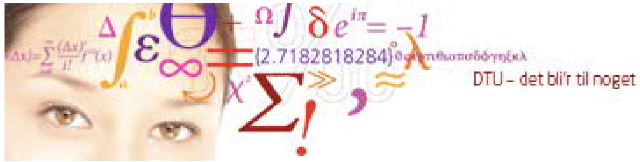
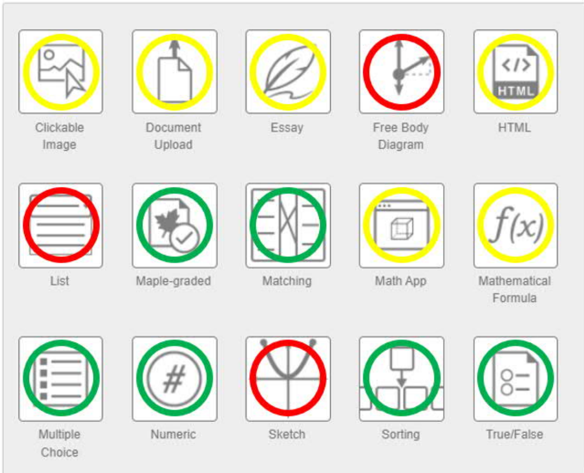


Karsten Schmidt and Ulrik Engelund Pedersen:

Möbius Assessment for both simple and more advanced purposes

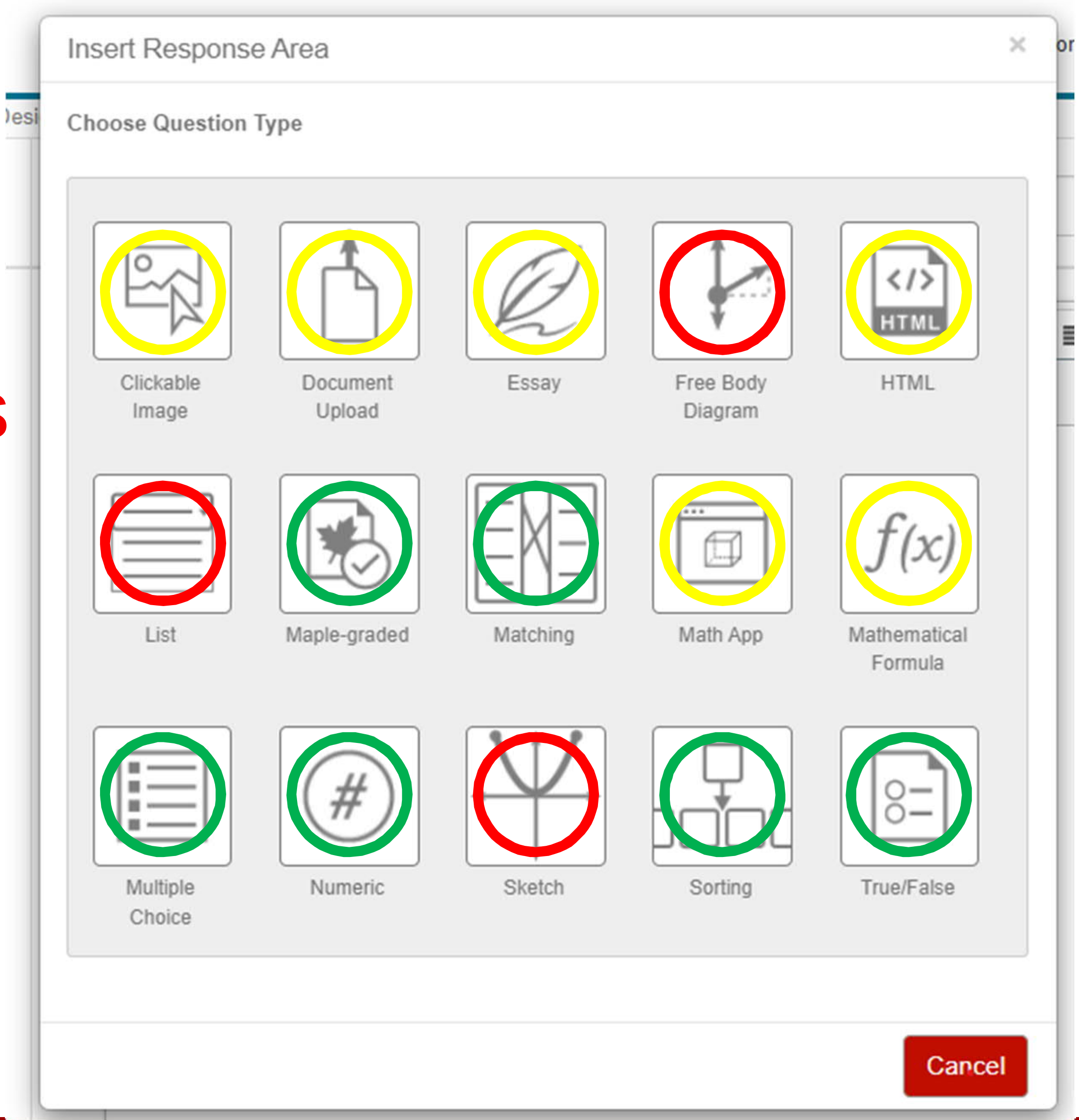


The 1st Northern e-Assessment Meeting, NTNU, June 1, 2023

Overview

1. Möbius Assessment (MA), product
2. Mathematics 1 at DTU (2001-2023):
   - Ideas, content and structure
   - Mathematical competencies
3. Three different ways of using MA
   - Weekly tests
   - Homework asssignments
   - Theme exercises
4. Conclusions

The options/ question types



Mathematics competence

The Danish KOM-report (2002, Mogens Niss et al.)

Definition:

Mathematics competence is defined as "the ability to understand, judge, do, and use mathematics in a variety of intra- and extra-mathematical contexts and situations where mathematics plays or could play a role".

The eight mathematical competencies

The Danish KOM-report (2002, Mogens Niss et al.)

1. Thinking mathematically
2. Reasoning mathematically
3. Posing and solving mathematical problems
4. Modeling mathematically
5. Representing mathematical entities
6. Handling mathematical symbols and formalism
7. Communicating in, with, and about mathematics
8. Making use of aids and tools

Adopted by The SEFI Mathematics Working Group 2013

Teaching elements, competencies and evaluation

1. **Lectures followed by group exercises**
   The mathematical topics are built linearly, step by step
   Competencies: 2, 3, 5, 6, 8
   Evaluation: A written exam at the end of each semester.
2. **Seven homework assignments**
   To unfold, explain and visualize the mathematical concepts and methods
   Competencies: 2, 3, 5, 6, 7, 8
   Evaluation: Corrected and graded by teaching assistents
3. **Seven theme exercises**
   After a new topic has been introduced, working with a simple model of a real-world problem.
   Competencies: 1, 3, 4, 8
   Evaluation: Quizzes
4. **A large 4 weeks group project-exercise**
   Several main topics brought together to investigate an authentic engineering problem.
   Competencies: 1, 2, 3, 4, 5, 6, 7, 8
   Evaluation: Group report and oral defense

Mathematics 1 – week by week

Autumn:

1. Intro to complex numbers
2. …continues
3. …continues
4. …continues
   HW 1 and Theme 1
5. Linear equations
6. Matrix-algebra
   HW 2 and Theme 2
7. Vector spaces
8. Linear transformations
   HW 3 and Theme 3
9. Function spaces
10. The eigenvalue problem
11. Linear diff. equations
12. Systems of diff. equations
13. HW 4 and Theme 4

Spring:

1. Functions of two variables
2. Taylor for several variables
3. Max/Min
   HW 5 and Theme 5
4. The Riemann integral
5. Surface integrals
6. Volume integrals
   HW 6 and Theme 6
7. Vector fields. Flux
8. Gauss's Theorem
9. Stokes's Theorem
   HW 7 and Theme 7
10. Big project execise
11. Big project execise
12. Big project execise
13. Big project execise

How can we further focus on specific competencies by fitting e-assessment into the learning process? Three examples: Weekly test, homework and theme exercises.

The weekly test

This idea is not new! Other recent reports:

Gaspar Martins, S. (2017). *Weekly online quizzes to a mathematics course for engineering students*

Rønning, F. (2017). *Influence of computer-aided assessment on ways of working with mathematics*

Feudel, F. & Unger, A. (2022). *Students' Strategic Usage of Formative Quizzes in an Undergraduate Course in Abstract Algebra*

Typical ideas/motivations:	**What we added:**
• To maintain the students' continuous engagement	• It is about the week's elementary skills (esp. competence 6)
• To give immediate feed back	• The test is held in the last class hour - with one attempt
• To save time on teaching or correction of homework	• Paper and pencil only (the test is monitored by TAs)
• The tasks can be randomized	• To increase the attendance and stage group work (learning by peers)
• The test is semi-mandatory	• A selection of the tasks will appear in the final written exam

Attendance to classes without TAs

Weekly schedule 2016-2023		
	Long Day	Short Day
08:00		Short Lecture
09:00		Group exercises without TAs
10:00	Lecture: Topic of the week	
11:00		Weekly Test
12:00		
13:00	Group exercises with TAs	
14:00		
15:00		

232 → Short Lecture
297 → Group exercises without TAs
238 → Lecture: Topic of the week
289 → Group exercises with TAs
Möbius Assessment → Weekly Test

(Red numbers indicate the number of attendees among the respondents)

Weekly test:

möbius

Preview

Given a matrix A and a diagonal matrix D :

$$A = \begin{bmatrix} 5 & -10 \\ 0 & 7 \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} 5 & 0 \\ 0 & 7 \end{bmatrix}$$

Provide an invertible matrix V such that: $V^{-1} \cdot A \cdot V = D$

$V =$

Grade | How Did I Do? | Refresh | Close

möbius

Preview

A surface $F_{\mathbf{r}}$ is given by the parametric representation:

$$\mathbf{r}(u,v) = (4\cos(v), 4\sin(v), u) \ , \ u \in [0,1] \ , \ v \in [0, 2\pi] \ .$$

A vector field $\mathbf{V}$ is given by the expression:

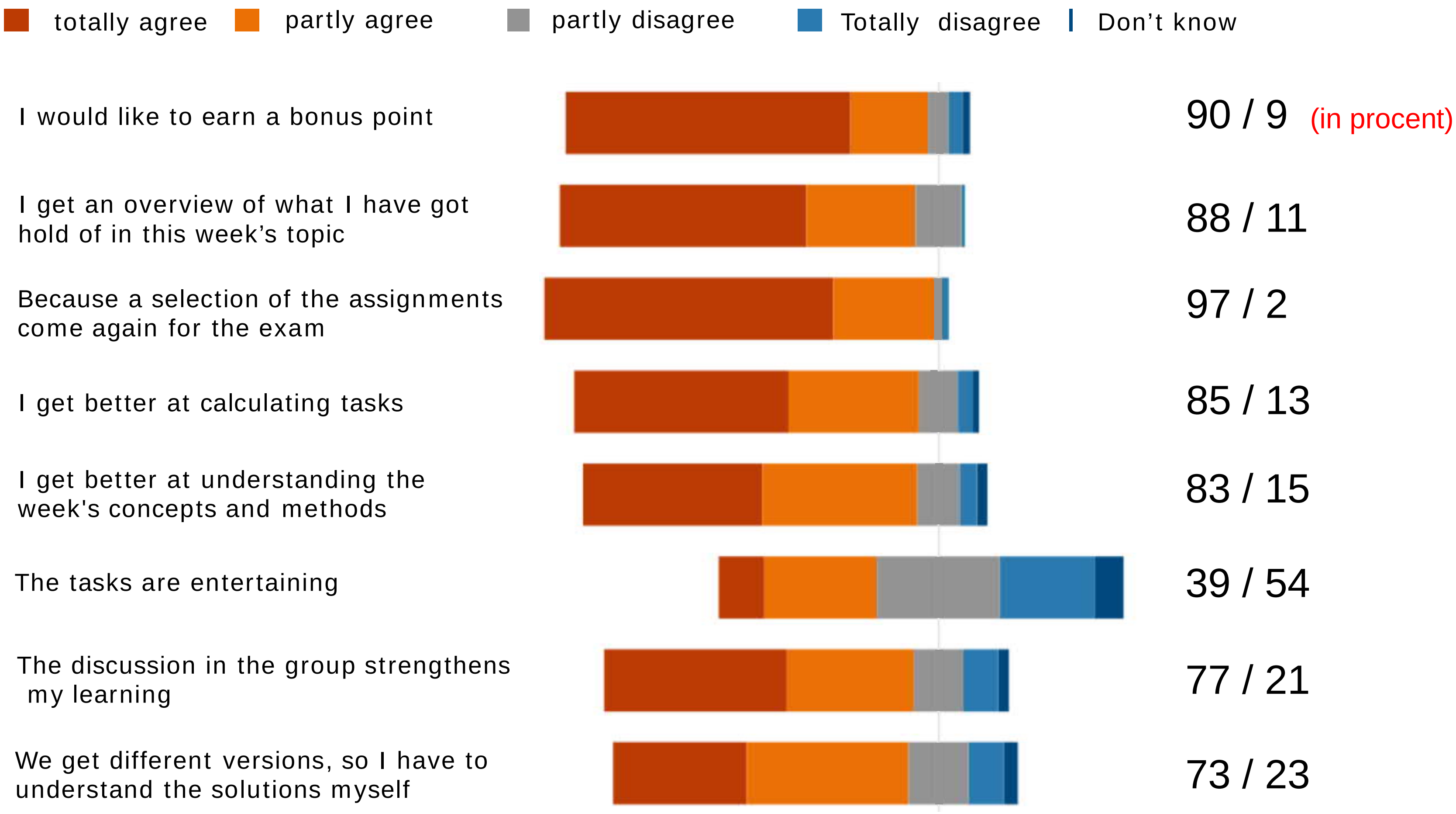
$$\mathbf{V}(x,y,z) = (3x, 3y, \exp(3x \cdot z)) \ .$$

Compute the integrand within the flux integral $\operatorname{Flux}(\mathbf{V}, F_{\mathbf{r}})$:

$\mathbf{V}(\mathbf{r}(u,v)) \cdot \mathbf{N}_F =$

Grade | How Did I Do? | Refresh | Close

My reasons for participating in the Weekly Test



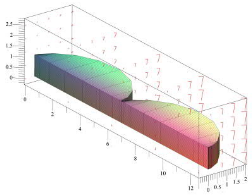
Quotes on the weekly test

Overall, the concept behind the weekly test is actually really good. Even if you get a lot of help from your peers present, you actually gain greater insight into the curriculum. At the same time, it is also good that it "forces" you to keep up every week. You cannot fail to do the tasks for Long / Short day, as otherwise you cannot pass the test.

I like to have part of the group exercises as paper and pencil, so I have to keep it up to date and understand some calculations better (...). But I think the weekly test is a really annoying tool for this purpose. I don't feel like I'm learning anything in stressed test form. Either I ask the others who hastily calculate it for me, otherwise, I memorize a method that I do not understand.

Typical homework hand-in style before

A clip from a paper:



Vi ønsker at bestemme den udadrettede enhedsnormalvektor til $\partial\Omega$

```
> M:=<vekdif(t(u,v,w),u)|vekdif(t(u,v,w),v)|vekdif(t(u,v,w),w)
  >;
```

$$M := \begin{bmatrix} 1-\cos(u) & 0 & 0 \\ v\sin(u) & 1-\cos(u) & 0 \\ -w\,v\sin(u) & -w\,(1-\cos(u)) & -v\,(1-\cos(u)) \end{bmatrix}$$

Vi bestemmer jacobi-funktionen ved at tage determinanten af vores matrix M

```
> Jacobi:=simplify(Determinant(M));
```

$$Jacobi := v\left(\cos(u)^3 - 3\cos(u)^2 + 3\cos(u) - 1\right)$$

Divergensens restriktion til det parametriserede område

```
> unapply(div(V)(x,y,z),[x,y,z]):
> DivV:=(x,y,z)->divV(vop(t(u,v,w))):'divV(x,y,z)'=divV(x,y,z)
  ;
```

$$divV(x, y, z) = 1 + 2\,y$$

Vi benytter Gauss's sætning

```
> flux:=abs(int(int(int(divV(vop(t(u,v,w)))*Jacobi,u=0..4*Pi),
  v=0..1),w=0..1));
```

$$flux := \frac{50}{3}\,\pi$$

Challenges:

- Long and confusing papers, difficult to correct and comment
- Bad training in dealing with usual mathematical notation
- Unsatisfactory explanations (Maple output regarded as the answer)

Typical homework hand-in style now

Solution:

- The first two problems are clean "Maple-problems" to be answered in MA
- The third problem is an "essay"-problem stressing competencies 2 and 7. Corrected and commented by TAs.
- Timeline: The three problems are published three weeks before deadline. MA opens two days before deadline with questions possibly twisted and randomized.

A clip from an essay:

Opgave 1 - Areal, omdrejningslegeme og rumintegral

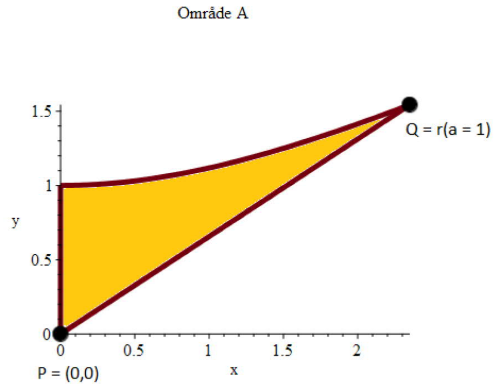
Lad *a* være et positivt tal. En kurve K_r i (x,z)-planen er givet ved parameterfremstillingen

$$r(u) = (2\sinh(u), \cosh(u)), \qquad u \in [0, a].$$

Vi betragter et begrænset, sammenhængende og afsluttet område *A* i (x,y)-planen hvis rand består af K_r, det rette linjestykke fra (0,0) til (0,1) og det rette linjestykke fra (0,0) til r(a)

a) Bestemmelse af en parameterfremstilling for A og den tilhørende Jacobi-funktion, samt ved hjælp heraf arealet af A.

For at give et overblik over området A, har jeg plottet dette nedenfor, hvor jeg har sat $a = 1$, sammen med punkterne $P = (0,0)$ og $Q = r(a)$.



Området *A* kan da beskrives som alle punkter beliggende på $\overrightarrow{PQ}$-vektoren, når *u* går fra 0 til *a*. En parameterfremstilling for området kan da udtrykkes som

$$A_r\colon\ r(u,v) = \begin{bmatrix} x \\ z \end{bmatrix} = v\overrightarrow{PQ} = v\begin{bmatrix} 2\sinh(u) \\ \cosh(u) \end{bmatrix} = \begin{bmatrix} 2v\sinh(u) \\ v\cosh(u) \end{bmatrix}, \qquad u \in [0,a]\ og\ v \in [0,1]$$

Jeg kan da finde Jacobifunktionen til denne parameterfremstilling ved at beregne den numeriske værdi af determinanten af den matrix der som kolonner har parameterfremstillingen $r(u,v)$ afledt mht. hhv. u og v:

$$Jacobi_r = |\det[r_u'(u,v),\ r_v'(u,v)]|$$

Homework set 6 (2023)

Parametrizations and Integration

06-03-23, shsp

Deadline: March 19 at 23:55.

Problem 1 and 2 are not for hand-in but must be answered in Möbius where they might be "twisted" compared to here. The Möbius version is open from Thursday March 16 at 12:00 in your DTU Learn group module.

Problem 3 is an essay assignment which you must upload as a .pdf file to your DTU Learn group module. Remember name and student number on your assignment.

In this homework set you must as general learning objectives demonstrate that you can

- *design fitting parametric representations of geometric objects in the 2D plane and in 3D space*
- *compute and utilize Jacobians corresponding to given parametric representations*
- *compute line, plane, surface, and volume integrals*
- *use these integrals to compute lengths, areas, volumes and masses*
- *use both elementary techniques and Maple techniques for integration*
- *create fitting illustrations using Maple*
- *write coherently and precisely and conduct simple mathematical reasoning*

Problem 1 Solid Region Between Two Planes. To be answered in Möbius

A bounded and closed region A in the 1st quadrant of the (x, y) plane is bounded by a hyperbola with the equation $x^2 - y^2 = 3^2$, a straight line with the equation $y = \frac{4}{5}x$, and the x axis.

a) Create a plot of A. Provide a parametric representation of A, and compute the corresponding Jacobian.

The homework set continues ⟼

b) Compute the area of A, and determine the centre of mass on A when the mass density function everywhere has a value of 1.

We now introduce two planes in (x, y, z) space that have the equations

$$z = -\frac{1}{4}y + 1 \quad \text{and} \quad z = -\frac{1}{8}y + \frac{1}{2}.$$

Let Ω denote the solid region that is located vertically above A between the two planes.

c) Provide a parametric representation $\mathbf{r}$ of Ω, and compute its corresponding Jacobian.

d) A function $f : \mathbb{R}^3 \to \mathbb{R}$ is given by the expression

$$f(x, y, z) = z.$$

Compute the volume integral

$$\int_{\Omega_r} f \, d\mu.$$

Problem 2 Surfaces and Solids of Revolution. To be answered in Möbius

We are given two real numbers a and b where $a \geq \sqrt{2}$ and $0 \leq b \leq \frac{\pi}{2}$.

A profile curve $\mathcal{K}$ in the (x, z) plane is given by

$$\mathcal{K} = \left\{ (x, z) \mid x = \ln(z) \, , \, z \in \left[\sqrt{2} \, , \, a \right] \right\}.$$

$\mathcal{K}$ is now rotated about the z axis from the directional angle $-b$ to the directional angle b in the (x, y) plane. This forms a surface of revolution F_{ab}.

a) Determine a parametric representation of $\mathcal{K}$ and of F_{ab}. Compute the Jacobian that corresponds to your parametric representation of F_{ab}.

b) Compute the surface integral

$$\int_{F_{ab}} \frac{z^2}{x} \, d\mu$$

using the values $a = 2\sqrt{2}$ and $b = \frac{\pi}{6}$.

The homework set continues ⟼

c) We now want the surface integral from the previous question increased by a factor of 10 by increasing either a or b. Create a plot of F_{ab} that includes both variants of the expanded surface. Which of them has the largest area?

Problem 3 Solid Region Bounded by Graph Surface. Essay, for pdf hand-in

A region A in the (x, y) plane is given by

$$A = \left\{ (x, y) \mid x \geq -1 \text{ and } 0 \leq y \leq 2 - \frac{1}{2}x^2 \right\}.$$

We consider the surface F that is the part of the graph of the function $h(x, y) = 2 - \frac{1}{2}x^2 - y$ that is located vertically above A.

a) Provide a parametric representation of A and of F, and compute the Jacobian that corresponds to your parametric representation of F.

b) Compute the surface integral of the function $f(x, y, z) = \dfrac{y^2}{\sqrt{2 + x^2}}$ over F.

c) Create a plot of F where its boundary curve ∂F is shown, and compute $\int_{\partial F} 6x \, d\mu$.

Let B denote the closed solid region that is located vertically between A and F.

d) Provide a parametric representation of B and compute the corresponding Jacobian.

e) Compute the volume of B.

End of problem sheet

Home assignments

möbius

Preview

Regarding HW6, problem 1a)

We are informed that a region A in the (x,y) plane can be parametrized by the parametric representation

$\mathbf{r}(u,v) = (v \cdot \cosh(u), v \cdot \sinh(u))$, $u \in \left[0, \operatorname{arccosh}\left(\frac{5}{3}\right)\right]$, $v \in [0,3]$.

1) Compute the value of the Jacobian function at the point that corresponds to $(u,v) = (\frac{1}{2}, \frac{9}{5})$.

$\mathrm{Jac}_{\mathbf{r}}(\frac{1}{2}, \frac{9}{5}) =$ ______

2) Where on region A does the Jacobian function reach its maximum value?

- ◯ Within the interior of A.
- ◯ At all points along the boundary of A that are found on the straight line $y = \frac{4}{5}x$.
- ◯ At all points along the boundary of A that are found on the x axis.
- ◯ At all points along the boundary of A that are found on the hyperbola segment.

Grade | How Did I Do? | Refresh | Close

möbius

Preview

Regarding HW6, problem 1d)

Compute the value of the following volume integral:

State the exact answer or state it with at least 2 decimals.

$\int_\Omega 5z \, \mathrm{d}\mu =$ ______

We now consider the new function:

$$g(x,y,z) = 2 \cdot x + 8 \cdot y \,.$$

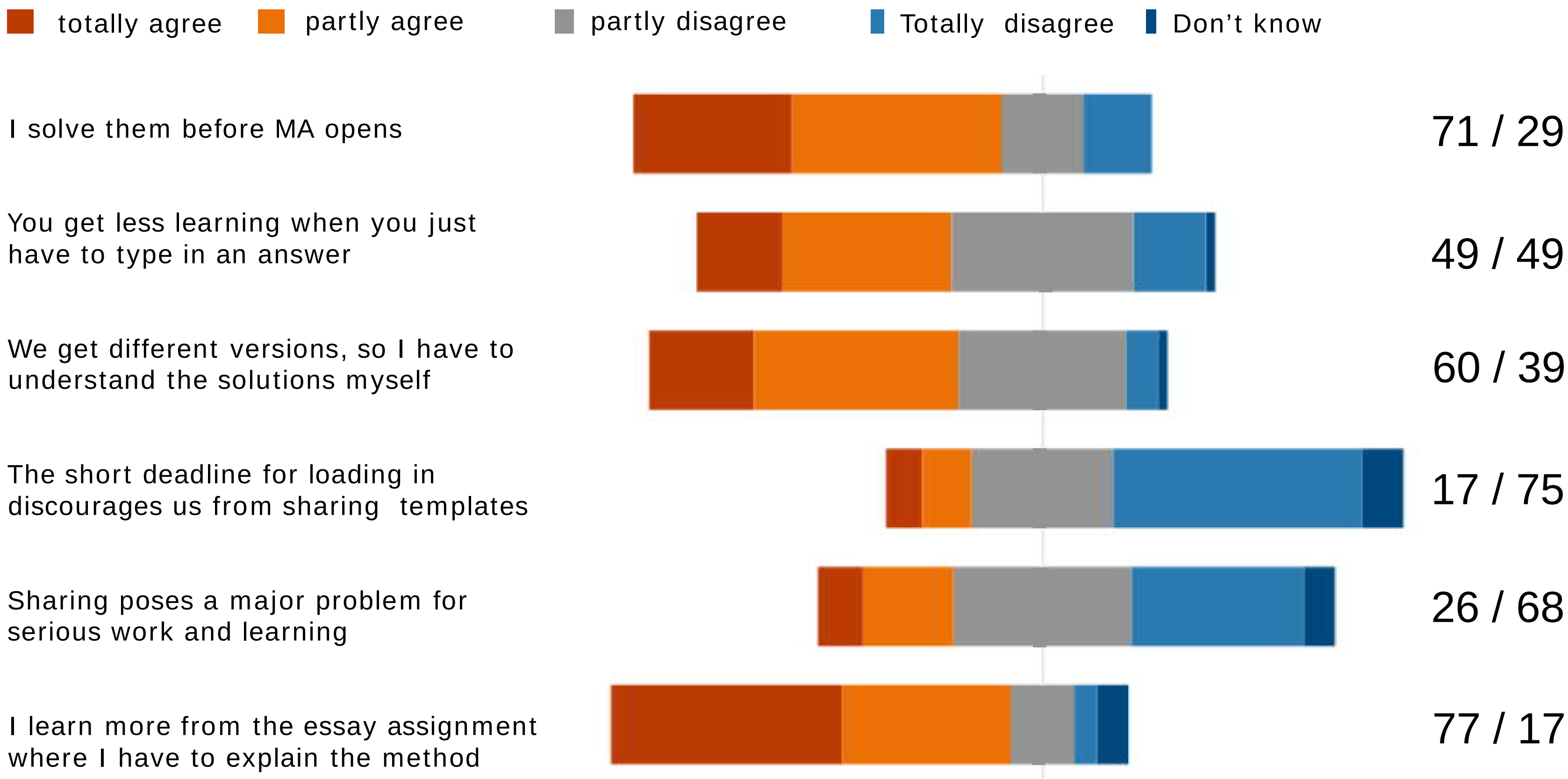
Compute the value of the following volume integral:

State the exact answer or state it with at least 2 decimals.

$\int_\Omega g \, \mathrm{d}\mu =$ ______

Grade | How Did I Do? | Refresh | Close

How I experience the two MA exercises in HW



Quotes on Homework

I think I learn equally from möbius and essay. It's great that we both get some tasks with free use of maple so that we can practice using it. At the same time, it is also nice to practice explaining the tasks. I therefore think it is a good mix

The Möbius part spoils the experience of the homework assignment. It doesn't make sense for a typo to count with, which it does not do when a human corrects in the same way. Doing the tasks in advance and they then get changed is an annoying method of setting things up (...) It feels a lot like wasted work.

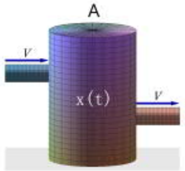
The fact that we all create and share templates with each other, I have felt has provided a good opportunity to discuss and compare different ways of solving the tasks. You get to train both the maple techniques, but also the understanding of the tasks and the ways you can solve them on.

Theme 4 (2022)

Theme Exercise 4

Coupled liquid containers

In all engineering sciences, there are real life problems that are modeled using systems of differential equations. Quite obvious examples are electrical networks or oscillations in building structures. In all cases, the mathematical challenge is the same: first it is about being able to model the given problem, thinking out of the box, so to speak, and then using the mathematical solution methods that are available. In this thematic exercise, we have chosen to describe the development of salt concentrations in systems of coupled liquid containers, and we confine ourselves to systems of linear first-order differential equation with constant coefficients. As an introduction to the modeling work, we first analyze a single (uncoupled) liquid container.



In the figure we see a container A with volume 100, which is full of water with added salt. At time $t = 0$ it opens up for an inflow into the container of saline water (same kind of salt) with constant concentration k. The inflow rate (which is measured in Liter / Sec, for example) is denoted v. An outflow occurs from the container at the same flow rate v.

The salt concentration in the container is denoted $x(t)$, it is the same everywhere in the container due to stirring. We want a functional expression for $x(t)$ so that at all times we know the salt concentration. But we can not extract that from the information given.

The Theme Exercise Continues⟶

We need to proceed indirectly. It turns out that it is possible to think of an expression for the velocity $x'(t)$ with which the salt concentration in the container is changing. It is the mathematical model of the problem, and once we have set it up, we can move on and hope that we can find solutions, ie. find $x(t)$. In the following initial problem, the model must be set up from scratch.

Problem 1 Exercise with setting up of mathematical model and solution

Let δt denote a small time interval, and answer the following questions:

a) How much water flows in / out of the container during δt ?

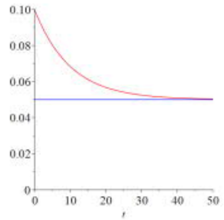
b) How much salt flows into the container during δt ?

c) How much salt flows out of the container during δt , assuming that the concentration of the outflowing water is $x(t)$?

d) How big is the change δx in the salt concentration of the container during δt (the volume of the container is set to 100 as mentioned above)?

e) On the basis of questions a) to d) find an expression for $x'(t)$, it is a first-order linear differential equation. Find the general solution to the differential equation as we set $k = \frac{1}{20}$ and $v = 10$.

f) The solution to the differential equation that satisfies the initial value condition that the salt concentration at time $t = 0$ is $\frac{1}{10}$, is here plotted using Maple with the color red:

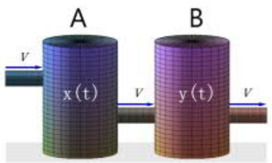


Find the functional expression for the conditional solution and describe in words how the salt concentration develops!

We now consider a system of two containers A and B, see the figure.

The Theme Exercise Continues⟶

Theme 4 (2022)



A has volume R_1 and B volume R_2, and they are filled with the same saline solution but with different concentrations. At time $t = 0$ they are connected by a pipe so that liquid flows from A to B at the flow rate v. There is also an inflow from the outside to A with clean water and a drain from B in both cases with the flow rate v. We want to find out how the salt concentration in each of the containers develops, ie. find an expression of the salt concentration $x(t)$ in A and the salt concentration $y(t)$ in B.

Problem 2 Mathematical model and solution for two containers

a) How big is the change δx in A's salt concentration over time δt? And how big is the change δy in B's salt concentration over time δt?

b) Now set up the mathematical model by, on the basis of question a) finding expressions for $x'(t)$ and $y'(t)$. Hint: The model is a system of two coupled first-order linear differential equations with constant coefficients, where the system matrix is a triangular matrix.

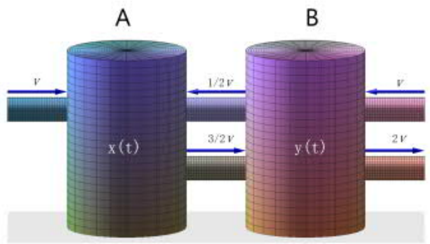
Now we fix $R_1 = 100, R_2 = 120$ and $v = 5$.

c) Set up the equation system in matrix form, and find eigenvalues and associated eigenspaces for the system matrix. Determine the general solution for the system in matrix form.

d) In the same plot, illustrate the solutions $x(t)$ and $y(t)$ to the system that fulfills the initial value condition $\begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 25 \\ 10 \end{bmatrix}$.

Compared to the previous scenario, a certain return from B to A is now added, and there is also a supply of clean water from the outside to tank B. See the relationship between the flow rates in the figure.

The Theme Exercise Continues⟶



Problem 3 Homogeneous and inhomogeneous system

a) How big is the change δx in A's salt concentration over time δt? And how big is the change δy in B's salt concentration over time δt?

b) Set up the mathematical model by, on the basis of question a) finding expressions for $x'(t)$ and $y'(t)$. Hint: The model is a homogeneous system of two coupled first-order linear differential equations with constant coefficients.

Now se fix $R_1 = R_2 = 100$ and $v = 10$.

c) Find (preferably with Maple's dsolve), and illustrate in the same plot the solutions $x(t)$ and $y(t)$ to the system that satisfy the initial value condition $\begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$. Express in words what happens to the saline solutions in A and B.

We now further assume that the liquid supplied from the outside to A and B is not pure water, but a saline solution. For A with concentration 1 and for B concentration 4. This results in an inhomogeneous system.

d) Find a particular solution for the system, by guessing a solution of the form

$$\mathbf{x}(t) = \begin{bmatrix} x_0(t) \\ y_0(t) \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} \text{ where } a, b \in \mathbb{R}.$$

Then determine the general solution to the inhomogeneous system using the structural theorem.

e) Find and illustrate in the same plot the solutions $x(t)$ and $y(t)$ to the system that fulfills the initial value condition $\begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$. Express in words what happens to the saline solutions in A and B.

The Theme Exercise Ends

DTU

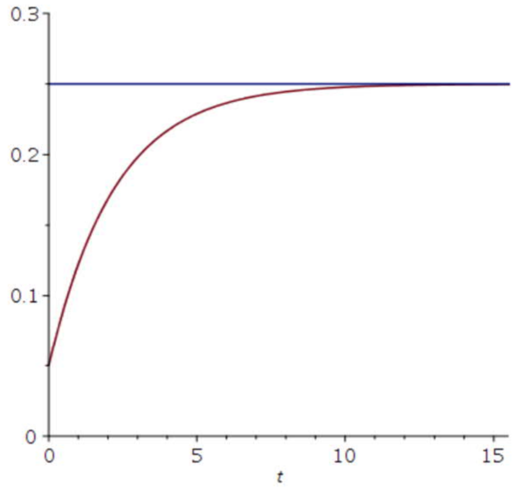
Theme 4

möbius

Preview

We consider a situation like the one in Theme 4, Problem 1f.

In the figure below a solution to the differential equation and its asymptote are plotted.



In addition it is given that the graph passes through the point $(5, \frac{1}{4} - \frac{1}{5}\,e^{-\frac{9}{4}})$.

Determine the inital value $x(0)$ and the values of the concentration k and the inflow velocity v:

$x(0) =$ Number

$k =$ Number

$v =$ Number

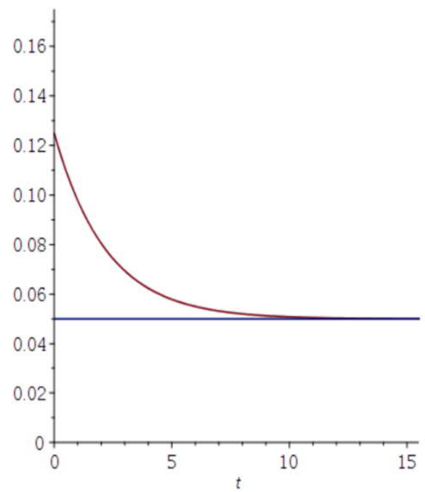
Grade | How Did I Do? | Refresh | Close

möbius

Preview

We consider a situation like the one in Theme 4, Problem 1f.

In the figure below a solution to the differential equation and its asymptote are plotted.



In addition it is given that the graph passes through the point $(6, \frac{1}{20} + \frac{3}{40}\,e^{-\frac{27}{10}})$.

Determine the inital value $x(0)$ and the values of the concentration k and the inflow velocity v:

$x(0) =$ Number

$k =$ Number

$v =$ Number

Grade | How Did I Do? | Refresh | Close

Theme 6 (2023)

Theme Exercise 6

Investigation of Architecture Using Integral Calculus

In this theme exercise we give a geometrical description of some well-known buildings designed by international architects. We consider surfacae areas , volumes and masses etc. We will need integration in one, two and three variables, but before we reach this state we need of course to establish the necessary parametric representations.

Problem 1 The Opera in Beijing

Enjoy initially the sight of the Opera in Beijing, designed by Paul Andreu and build in 2007:

About the building one can on the internet read: *It is a curved building, with a total surface area of 149, 500 square meters, that emerges like an island at the center of a lake. The titanium shell is in the shape of a super ellipsoid with a maximum span of 213 meters, a minimum span of 144 meters and a height of 46 meters).* [1]

The Theme Exercise Continues⟶

In the following we assume that the opera building can be modeled as a standard half-ellipsoid.

a) Determine the area of the base of the opera together with its volume using the given information about its maximum and minimum diameter in the base and the height.

b) Assess the information about the total surface area of the building.

c) We assume that the surface shell weighs 20 kg pr. m^2 at the bottom and that it decreases continuously with a fixed percentage per unit height until it at the top weighs 10 kg pr. m^2. Determine the mass of the surface shell.

Problem 2 Tycho Brahe Planetarium

Then we are back in Copenhagen and look at the Thyco Brahe Planetarium, designed by the Danish architect Knud Munk and built 1988-89. You notice the beautiful oriental-inspired patterns on the facade, but this is outside the scope of our problem.

The planetarium appears as a cylinder of revolution cut off by an inclined plane. Thereby, the roof forms an ellipse – perhaps a reference to the planets' orbits around the sun. From Wikipedia it appears that the building is 38 meters high, it must be where it is

The Theme Exercise Continues⟶

Theme 6 (2023)

highest. From photos we can guess that it is half as high where it is lowest. A selection of the building drawings [2] is available on the Art Library's website. If you look closely at them you will find the radius of the cylinder 12.8 meters and the thickness of the outer wall 0.30 meters.

a) Establish a parametric representation for the planetarium's roof, and determine from that the area of that roof. Hint: The roof can be perceived as a *graph surface* lying over a circular disk in the (x, y) plane with the center at the origin.

b) Determine a parameter representation for the outer wall of the planetarium, and determine the volume of the outer wall.

c) We assume that the stones in the outer wall are massive at the bottom and more porous upwards. Thereby, the density of the stone of the outer wall is 2.3 kg / l at the ground, but it decreases linearly so that it is 1.8 kg / l at the top of the building. Determine the mass of the total outer wall.

Problem 3 St. Mary Axe in London

Now we move to London and consider Norman Forster's glass-skyscraber St. Mary Axe (popularly known as The Gherkin) from 2004:

Here we shall investigate some of the properties of the building, based on a parametrization of its surface, that is a surface of revolution. The building is thought to be imbedded in an ordinary (x, y, z)-coordinate system such that the axis of revolution

The Theme Exercise Continues⟶

of the building runs along the z-axis. We assume that the profile curve, placed in the (x, z)-plane, can be modeled by a function of the form

$$x = f(z) = \sqrt{az^2 + bz + c} \text{ hvor } a, b, c \in \mathbb{R}.$$

a) Determine a, b and c from the following information that can be found on the internet, see e.g. [3] and [4]: 1) The building is 180 meters heigh, 2) radius at the bottom is 24 meters and 3) the building reaches its maximum width at the height 66 meters.

b) State a parametric representation for the surface of the building and illustrate.

c) Determine the area of the surface.

It is given in the said material that in total 8358 tons of steel has been used in the building. Let $m(z)$, $z \in [0, 180]$, denote the average mass density of steel in the building at the height z, measured in $tons/m^3$. It is assumed that $m(z)$ decreases linearly with the height such that it at the top is half as large as it is at the bottom.

d) Determine $m(0)$.

Theme 6

möbius

Preview

Regarding Theme 6, Problem 2c).

State the mass M of the total outer wall.

Enter a number without decimals.

$M =$ [ ] ton

We now change the density of the stones of the outer wall to 2.7 kg/L at ground level, but it decreases linearly so that it at the top of the bouilding is 2.1 kg/L.

Determine the new mass M_{new} of the total outer wall.

Enter a number without decimals.

$M_{\text{new}} =$ [ ] ton

Grade | How Did I Do? | Refresh | Close

möbius

Preview

Regarding Theme 6, Problem 2c).

We now assume instead that the density decreases by a fixed percentage per meter from 2.3 kg/L at the bottom to 1.8 kg/L at the top.

By what percentage r does the density decrease per meter?

Enter a number with 2 decimals.

$r =$ [ ] %

State the mass M of the entire outer wall with this new mass distribution.

Hint: Consider your integration order if Maple has difficulties calulating the result.

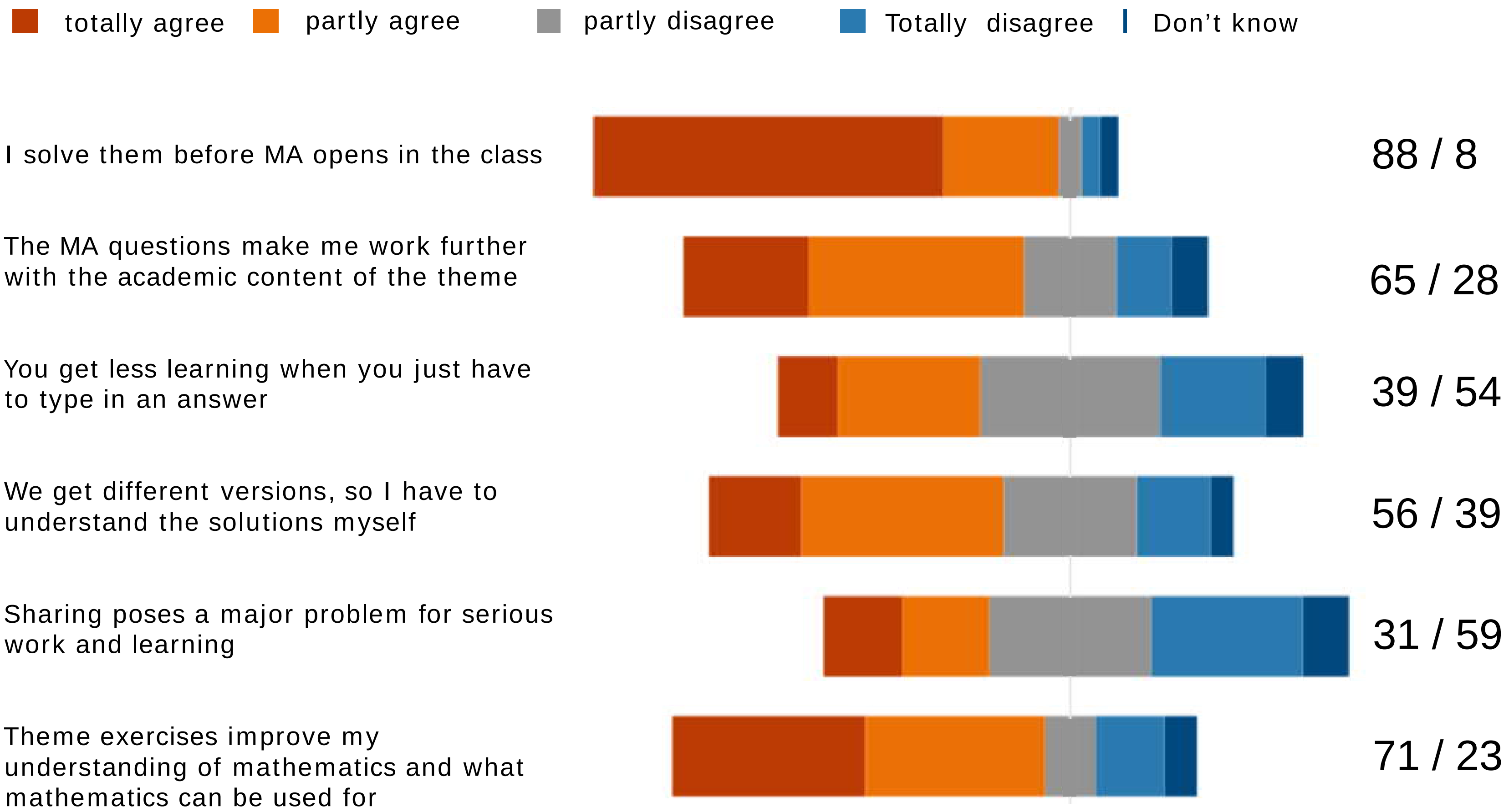
Enter the answer without decimals.

$M =$ [ ] ton

Grade | How Did I Do? | Refresh | Close

DTU

My experience and use of theme exercises



Quotes on theme exercises

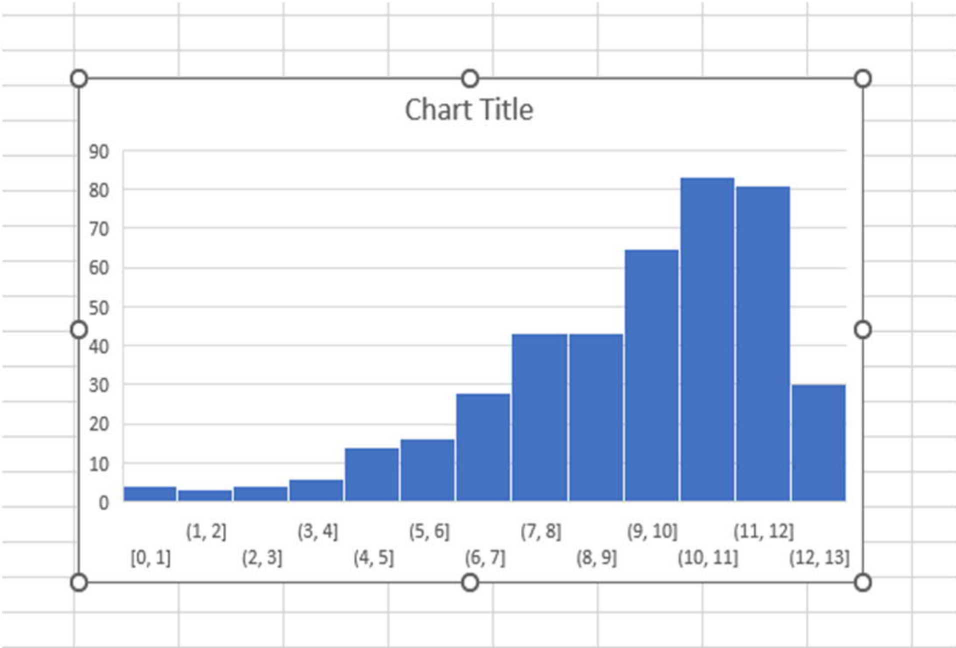
I'm a big fan of theme. For me, it helps me a lot to round off the different topics we have had on Math 1. Sitting with 5 other fellow students to solve and discuss the tasks together works super well.

The theme exercises feel a bit meaningsless, as they only contribute to understanding, and do not help us directly to the final goal, which is undoubtedly the exam.

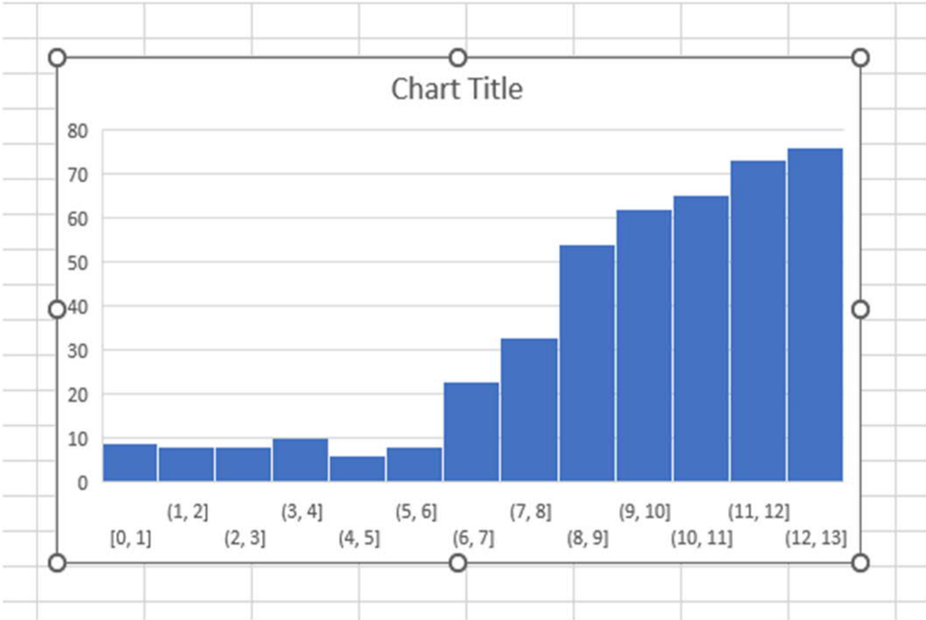
I do not think the result of the theme exercises is a correct representation of the individual's level in the subject. Often "cheat sheets" with answers to all tasks are handed out, and it is often those who 'know someone' who do it best.

Theme 4 scheme group A

2021



2022



Conclusions

By e-assassment we can:

1. Change study behavior:
   - Increase attandence
   - Make the students work continously and harder
2. Stage fruitful group work
3. Support several compentencies (but not all)

Challenges to be studied further:

1. The stress factor (positive - negative)
2. The sharing problem (positive – negative)
3. Motivation (positive – negative)

Thank you for attention!